On the Valuation of Uncertainty in Welfare Analysis

Jean-Paul Chavas and Daniel Mullarkey

This article develops a general model of private and public choice under temporal uncertainty. The model incorporates the effects of risk preferences and the prospect of future learning into both the individual and aggregate valuations of public projects. The analysis provides new insights on individual valuation, its implications for benefit–cost analysis and the characterization of Pareto-efficiency under uncertainty. It also resolves some of the confusion in the option value and quasi-option value literature.

Key words: information, option value, quasi-option value, risk, uncertainty, welfare.

Private and public decisions typically involve many sources of uncertainty, including uncertainty about prices, income, technology, environmental quality, health, etc. Temporal uncertainty is also common, where some of the risk[1] gets resolved over time due to learning. Additionally, private and public decisions often exhibit some form of irreversibility, implying that current choices can affect the ability of decision makers to be flexible and to adjust their future choices in response to forthcoming information. When uncertainty and irreversibility are both present, welfare analysis gains complexity and importance. Although considerable progress has been made since Weisbrod's seminal article, a better understanding of the roles of uncertainty and irreversibility in welfare analysis would improve our ability as economists to communicate the importance of risk in project evaluation to public decision makers. The objective of this article is to clarify and enhance the understanding of welfare analysis under uncertainty and irreversibility, with particular attention paid to the role of temporal learning at both the private and public levels.

As an example illustrating the joint effects of uncertainty and irreversibility, consider the case of groundwater pollution. The uncertainty may relate to the pollution source (e.g., accidental spill), to the movement of the water pollutants over time and space, to the exposure of some human population to these pollutants, and to the short- and long-term effects of the pollution on human health. At least two forms of irreversibility may be present: irreversibility of groundwater pollution if the movement and dilution of pollutants in the groundwater are slow; and irreversibility of health effects if the pollutants have long-term effects on human health (including death). This example illustrates that irreversibility can be relevant at both the social level (e.g., the long-term human exposure to a toxic site) and the individual level (e.g., chronic health effects). This suggests the need for a careful analysis of the joint welfare effects of uncertainty and irreversibility at both the social level and the individual level (Bromley and Segerson).

Starting with Weisbrod, the valuation of uncertainty has focused on the concept of "option value." Two main bodies of literature have emerged. One approaches the concept of option value (OV) as a risk premium associated with possible risk aversion (Byerlee; Schmalensee; Bishop 1982, 1988;

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[1] Throughout the article, we use the terms "risk" and "uncertainty" interchangeably. Thus, we do not rely on the Knightian distinction between risk (where probabilities are available to guide choice) and uncertainty (where information is too imprecise to be summarized adequately by probabilities). Our analysis is consistent with a Bayesian approach, where individual beliefs about uncertainty can be represented by a subjective probability measure, and learning over time takes place through Bayesian updating of these probabilities.

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Smith 1983, 1987a; Freeman 1985; Hartman and Plummer). This literature explores the idea that an individual's risk preferences can affect his/her valuation of a project. It concludes that, in general, the risk aversion premium can take any sign, and that it should be incorporated in the benefit–cost analysis of a project.

The second concept of option value, sometimes called quasi-option value (QOV), focuses on the impact of current decisions on the future use of information (Arrow and Fisher; Henry 1974a, 1974b; Conrad 1980; Smith 1983; Miller and Lad; Fisher and Hanemann 1986a, b, 1987, 1990; Hanemann 1989). Many of the models in this literature assume risk neutrality; the rest do not address risk preferences explicitly. In a temporal model, learning is valuable in future decisions if a reversible decision is made in the first period. If an irreversible choice is made, future decisions can no longer respond to new information. Choosing the reversible alternative in the first period preserves the decision maker's flexibility to respond to future learning. The QOV literature shows that failure to take this flexibility into account leads to inefficient decisions that are systematically biased toward the irreversible alternative.[2]

Considerable progress has been made in defining OV and QOV, and in establishing the differences between them. Additionally, Graham (1981, 1992) has provided a powerful framework within which to study welfare analysis under uncertainty. The 1981 article introduced the idea that state-dependent payment schemes could be used to capture benefits above and beyond the state-independent payments (option price) discussed in the OV literature. The 1992 article provided a net benefit function and related criteria for identifying Pareto-efficient projects.

Although he did not pursue it, Graham (1992, p. 836) mentions that his net benefit function could be used to decompose the benefits of a project into "pure risk distribution gains" and "pure output gains." Given the confusion in the literature on welfare analysis under uncertainty, including misinterpretation of Graham's work, we feel this decomposition would be informative. We explore the general case where private and public decisions coexist, and show how interactions between these decisions influence the valuation of risk. We demonstrate that even if a public project is reversible, it may still have irreversible effects on private decisions that would affect its welfare evaluation.

This article develops a welfare criterion that is dual to Graham's Pareto-efficiency criterion. It then decomposes the total value of a project into one risk-free output component and three risk components: (1) the individual value of learning; (2) the individual cost of risk aversion; and (3) the social valuation of information. This decomposition allows us to illustrate the concepts behind OV and QOV within one model of total value, which to our knowledge has not been done,[3] and hopefully resolves much of the confusion in the OV and QOV literatures. Graham (1981) showed that a welfare measure that neglects the welfare gains attributed to state-dependent financing would be a lower bound on a welfare measure that allows for such gains. The decomposition of value presented here extends Graham's result by formally showing that a welfare measure that neglects the state-dependent provision of public goods is also a lower bound on a more inclusive measure that includes the risk benefits of *both* state-dependent project financing and state-dependent provision of public goods (although he only briefly references it, Graham's (1992) model is flexible enough to analyze state-dependent provision of public goods).

Primal and Dual Pareto-Efficiency Criteria

In this section, we develop a Pareto-efficient criterion for project evaluation. We also establish the duality relationships between our (primal) approach and Graham's (1992) dual formulation of the net benefit criterion. For simplicity, consider a two-period model.[4]

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[2] Hanemann (1989) shows that the assumption of perfect information can be relaxed without changing the conclusions as long as the other assumptions are maintained. Additionally, he discusses the conditions under which QOV increases with increasing uncertainty.

[3] Chavas, Bishop, and Segerson examine the effect of learning on welfare measurements. However, they do not investigate its implications for Pareto-efficiency. Also, Fisher and Hanemann (1987) discuss both passive and active learning under irreversibility and risk neutrality. We extend their analysis by considering the role of risk aversion, by examining the more general issue of dynamic flexibility, and by establishing explicit linkages with Pareto-efficiency.

[4] Limiting our analysis to a two-period model greatly simplifies the presentation, and captures the essence of temporal uncertainty and learning. All the arguments presented below can be easily extended to a more general T-period planning.

Let $\mathbf{I} = \{1, \ldots, n\}$ denote the set of n individuals facing a public project under uncertainty. The "project" may be a policy change (e.g., regulations), policy action (e.g., enforcement of regulations), or an actual physical project (e.g., building a dam). The project may be partially or fully irreversible. Define the project as involving two types of decisions: the production of public goods generated by the project, and the financing of the project. In period t, let $\mathbf{z}_t = (\mathbf{z}_{tp}, \mathbf{z}_{tf})$, where $\mathbf{z}_{tp}$ denotes the vector of public goods generated by the project and $\mathbf{z}_{tf} = (\mathbf{z}_{t1f}, \ldots, \mathbf{z}_{tnf})$ denotes the financing of the project, with $\mathbf{z}_{tif}$ being the amount of money *actually paid* (or received, if negative) by the ith individual. Let $\mathbf{z}_p = (\mathbf{z}_{1p}, \mathbf{z}_{2p})$ and $\mathbf{z}_f = (\mathbf{z}_{1f}, \mathbf{z}_{2f})$. In general, both $\mathbf{z}_p$ and $\mathbf{z}_f$ can be state dependent, and their elements should be interpreted as decision rules.

To characterize the information available for public decision making, consider three sets of random variables: $(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}}, \mathbf{e}_2^{\mathrm{I}})$, where $\mathbf{e}_0^{\mathrm{I}}$ is the vector of random variables observed by the policy maker before the first-period decisions $\mathbf{z}_1$ are made, $\mathbf{e}_1^{\mathrm{I}}$ is the vector of random variables observed by the policy maker after the first-period decisions $\mathbf{z}_1$ but before the second-period decisions $\mathbf{z}_2$, and $\mathbf{e}_2^{\mathrm{I}}$ is the vector of random variables observed after the second-period decisions. First-period decisions $\mathbf{z}_1$ can depend on the signals $\mathbf{e}_0^{\mathrm{I}}$, but not on $(\mathbf{e}_1^{\mathrm{I}}, \mathbf{e}_2^{\mathrm{I}})$. Second-period decisions $\mathbf{z}_2$ can depend on $(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})$, but not on $\mathbf{e}_2^{\mathrm{I}}$.[5] With active public learning, the $\mathbf{e}_1^{\mathrm{I}}$ information available for second-period decisions $\mathbf{z}_2$ depends on first-period decisions $\mathbf{z}_1$. Passive learning is a special case in which $\mathbf{e}_1^{\mathrm{I}}$ does not depend on $\mathbf{z}_1$. The collective decisions amount to choosing *decision rules* $\mathbf{z}_1(\mathbf{e}_0^{\mathrm{I}})$ and $\mathbf{z}_2(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})$, which map the observed signals into actual decisions.

The feasibility of the project is defined in terms of fiscal as well as technical feasibility.[6] *Technical feasibility* for the public goods $\mathbf{z}_p$ is characterized by the feasible set $\mathbf{Z}$: $\mathbf{z}_p \in \mathbf{Z}$. This includes as special cases models of irreversible development that have been at the heart of the quasi-option value literature (e.g., Arrow and Fisher; Hanemann, 1989). For example, in Hanemann's (1989) model, if $\mathbf{z}_{tp}$ is the proportion of irreversible development at time t, then $(\mathbf{z}_{1p}, \mathbf{z}_{2p}) \in \mathbf{Z} = \{[0,1], [0, 1 - \mathbf{z}_{1p}]\}$. Assume the existence of a riskless asset yielding a sure rate of return r between time $t = 1$ and $t = 2$, such that each individual can borrow or lend any amount of money at the riskless rate r. Let $C(\mathbf{z}, \mathbf{e}^{\mathrm{I}}) = C_1(\mathbf{z}_1, \mathbf{e}_0^{\mathrm{I}}) + \beta C_2(\mathbf{z}_1, \mathbf{z}_2, \mathbf{e}^{\mathrm{I}})$ denote the present value of aggregate cost associated with the project, where $C_1(\mathbf{z}_1, \mathbf{e}_0^{\mathrm{I}})$ is the cost incurred at time $t = 1, \beta = 1/(1 + r)$ is the discount factor, and $\beta C_2(\mathbf{z}_1, \mathbf{z}_2, \mathbf{e}^{\mathrm{I}})$ is the present value of the cost incurred at time $t = 2$. This allows for uncertainty in the cost of the project, as reflected by the random vector $\mathbf{e}^{\mathrm{I}} = (\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})$. Note that $C(\mathbf{z}, \mathbf{e}^{\mathrm{I}})$ can include various transaction costs (including information costs) associated with the actual implementation of the project. *Fiscal feasibility* is then characterized by the aggregate budget constraint

$$(1) \qquad \sum_{i \in \mathbf{I}} [\mathbf{z}_{1if}(\mathbf{e}_0^{\mathrm{I}}) + \beta\, \mathbf{z}_{2if}(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})] \geq C(\mathbf{z}, \mathbf{e}^{\mathrm{I}}).$$

Equation (1) states that the present value of *actual payments* made by all n individuals must cover the present value of the cost of collective action in each state. This allows the financing of the project to take place at time $t = 1$, as well as at time $t = 2$ after the random vector $\mathbf{e}_1^{\mathrm{I}}$ is observed. Because at least partial information becomes available as to which state occurred in period 1, period-two financing $\{\mathbf{z}_{2if}(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})\}$ can be designed to generate efficiency gains through risk redistribution.

The problem is thus to choose decision rules for the financing scheme $\{\mathbf{z}_{tif}\}$, and for the levels of public goods (or bads) $\mathbf{z}_{tp}$ to provide in each period, given the information publicly available at the time of the decisions. Let $P_i^s(y_i, \mathbf{z}_{1if} - \beta \mathbf{z}_{2if}, \mathbf{z}_p)$ denote the ith individual's valuation of the project, where y_i is the ith individual's exogenous income.

PROPOSITION 1: *Assuming that the n individuals are nonsatiated in income, if an allocation is Pareto-efficient, it satisfies the optimization problem*

$$(2) \qquad W^1(\alpha) = \underset{\substack{\mathbf{z}_1(\mathbf{e}_0^{6\mathrm{I}}) \\ \mathbf{z}_2(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})}}{\mathrm{Max}} \left\{ \sum_{i \in \mathbf{I}} \alpha_i P_i^s(y_i, \mathbf{z}_{1if} - \beta \mathbf{z}_{2if}, \mathbf{z}_p) : \text{s.t.}\, \mathbf{z}_p \in \mathbf{Z} \textit{ and } (1) \right\}$$

[5] As will be discussed below, public decisions and public learning can also be dependent on private decisions and private learning, and vice versa.

[6] The technical and fiscal feasibility constraints defined here are analogous to the production transformation set in Graham (1992).

where $\alpha_i > 0$ is a welfare weight for the ith individual satisfying the normalization restriction $\sum_{i\in\mathbf{I}} \alpha_i = 1$.

Proof. Consider a feasible allocation $\mathbf{z}$ that does not satisfy (2). It means that there exists another feasible allocation $\mathbf{z}'$ that increases the selling price P_i^s for at least one individual. Under nonsatiation, $\mathbf{z}'$ can make this individual better off without making anyone else worse off. This contradicts the Pareto-efficiency criterion.

Denote the optimal decision rule for $\mathbf{z}$ in (2) by $\mathbf{z}^*(\mathbf{e}^{\mathrm{I}}, \alpha) = (\mathbf{z}_1^*(\mathbf{e}_0^{\mathrm{I}}, \alpha), \mathbf{z}_2^*(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}}, \alpha))$, where $\alpha = \{\alpha_i\} = (\alpha_1, \ldots, \alpha_n)$. Proposition 1 states that (2) is a necessary condition for Pareto-efficiency. In addition, it can be shown that (2) is also sufficient for all allocations that are not on the boundary of the consumption set (e.g., see Luenberger, p. 231). Then, $\mathbf{z}^*(\mathbf{e}^{\mathrm{I}}, \alpha)$ is *Pareto-efficient.* Letting $U_i^*(\alpha) = U_i(y_i, \mathbf{z}_{1if}^* - \beta\mathbf{z}_{2if}^*, \mathbf{z}_p^*)$, where $U_i(\cdot)$ is the ith individual's indirect utility function, it follows that the *Pareto-utility frontier* is traced by $U^*(\alpha) = \{U_i^*(\alpha), i \in \mathbf{I}\}$ as α takes all possible positive values in the unit simplex. The Pareto-utility frontier $U^*(\alpha)$ defines the set of individual expected utilities that cannot be Pareto-improved upon by any feasible change in either financing or the levels of public goods provided.

How does this approach relate to Graham's (1992) analysis? Consider the case of discrete random variables, where the realized values of $\mathbf{e}^{\mathrm{I}}$ generate m mutually exclusive observed states of nature.[7] The state-dependent decisions $\mathbf{z}(\mathbf{e}^{\mathrm{I}})$ can then be written as: $\{\mathbf{z}_j, j \in \mathbf{J}\}$, where $\mathbf{z}_j$ is the $\mathbf{z}$ decision made under the jth state of nature, and $\mathbf{J} = \{1, \ldots, m\}$ is the set of all possible states of nature. Let $C_j(\mathbf{z}_j)$ denote the cost of the project under the jth state of nature.

PROPOSITION 2: *Under nonsatiation, assume that the set $\mathbf{Z}$ is convex, that the functions $U_i(y_i, \mathbf{z})$ are quasi-concave, that the functions $C_j(\mathbf{z}_j)$ are convex, and that there exists a feasible $\mathbf{z}$ where equation (1) holds with a strict inequality. Then Pareto-efficient allocations correspond to*

$$(3)\quad M(U^0) = \underset{\gamma\geq 0}{\mathrm{Min}}\, \underset{z}{\mathrm{Max}} \Big\{ \sum_{j\in\mathbf{J}} \gamma_j \Big[\sum_{i\in\mathbf{I}} (\mathbf{z}_{1ifj} + \beta\mathbf{z}_{2if}) - C_j(\mathbf{z}_j) \Big] : U_i^0 = U_i(y_i - \mathbf{z}_{1if} - \beta\mathbf{z}_{2if}, \mathbf{z}_p); \; \mathbf{z}_p \in \mathbf{Z}; \textstyle\sum_{j\in\mathbf{J}} \gamma_j = 1 \Big\},$$

where $U^0 = (U_1^0, \ldots, U_n^0)$ is chosen such that $M(U^0) = 0$, $M(U^0)$ being Graham's (1992) net benefit criterion.

Proof. From equation (2) in Proposition (1), Pareto-efficiency can be written as

$$(4a)\quad 0 = \underset{z}{\mathrm{Max}} \Big\{ -W^I(\alpha) + \sum_{i\in I} \alpha_i P_i^s(y_i, \mathbf{z}_{1if} - \beta\mathbf{z}_{2if}, \mathbf{z}_p) : \text{s.t. } \mathbf{z}_p \in \mathbf{Z} \text{ and } (1) \Big\}.$$

Note that a feasible point on the Pareto-utility frontier corresponds to $W^{\mathrm{I}}(\alpha) = \sum_{i\in\mathbf{I}} \alpha_i P_i^s(y_i, \mathbf{z}_{1if} - \beta\mathbf{z}_{2if}, \mathbf{z}_p)$, or equivalently to $\{U_i(y_i - \mathbf{z}_{1if} - \beta\mathbf{z}_{2if}, \mathbf{z}_p) = U_i^*(\alpha), i \in N\}$. It follows that (4a) can be alternatively written as

$$(4b)\quad 0 = \underset{z}{\mathrm{Max}} \Big\{ -W^{\mathrm{I}}(\alpha) + \sum_{i\in I} \alpha_i P_i^s(\cdot) : \text{s.t.} (1), \mathbf{z}_p \in \mathbf{Z}, \text{ and } U_i(y_i - \mathbf{z}_{1if} - \beta\mathbf{z}_{2if}, \mathbf{z}_p) = U_i^*(\alpha), i \in \mathbf{N} \Big\}.$$

The Lagrangean associated with (4b) is $L(\mathbf{z}, \gamma) = -W^{\mathrm{I}}(\alpha) + \sum_{i\in\mathbf{I}} \alpha_i P_i^s(\cdot) + \sum_{j\in\mathbf{J}} \gamma_j [\sum_{i\in\mathbf{I}} (\mathbf{z}_{1ifj} + \beta\mathbf{z}_{2ifj}) - C_j(\mathbf{z}_j)]$, where $\gamma_j \geq 0$ are Lagrange multipliers associated with the budget constraint (1) under the jth state of nature. Under regularity conditions (including the convexity conditions stated in Proposition 2; see Takayama, p. 75), equation (4b) can be written as the saddle-point of the Lagrangean $L(\mathbf{z}, \gamma)$, yielding

$$(4c)\quad 0 = \underset{\gamma\geq 0}{\mathrm{Min}}\, \underset{\mathbf{z}}{\mathrm{Max}} \Big\{ -W^{\mathrm{I}}(\alpha) + \sum_{i\in\mathbf{I}} \alpha_i P_i^s(\cdot) + \sum_{j\in\mathbf{J}} \gamma_j \Big[\sum_{i\in\mathbf{I}} (\mathbf{z}_{1ifj} + \beta\mathbf{z}_{2ifj}) - C_j(\mathbf{z}_j) \Big] : \text{s.t. } \mathbf{z}_p \in \mathbf{Z}, \text{ and } U_i(y_i - \mathbf{z}_{1if} - \beta\mathbf{z}_{2if}, \mathbf{z}_p) = U_i^*(\alpha), i \in \mathbf{N} \Big\}$$

[7] For simplicity, the analysis presented in proposition 2 ignores learning. This is consistent with the model presented by Graham (1992). Incorporating learning in proposition 2 could be done using information partitions, with the partition associated with period 2 being finer than the one associated with period 1.

which, at the optimum, satisfies the complementary slackness condition $\sum_{j\in\mathbf{J}}\gamma_j[\sum_{i\in\mathbf{I}}(\mathbf{z}_{1ifj}+\beta\mathbf{z}_{2ifj})-C_j(\mathbf{z}_j)]=0$. Let $U_i^0=U_i^*(\alpha)$, $i\in\mathbf{N}$. Because $W^1(\alpha)\geq(=)\sum_{i\in\mathbf{I}}\alpha_i P_i^s(\cdot)$ for feasible (Pareto-efficient) allocations, (4c) yields the desired results, subject to the normalization rule $\sum_{j\in\mathbf{J}}\gamma_j=1$.

Proposition 2 gives the representation of Pareto-efficiency developed in Graham (1992). Equation (3) highlights the role played by the γ's, which can be interpreted as shadow prices for contingent claims. Braden and Kolstad, Smith (1987b) and Graham (1992) have stressed the importance of these shadow prices in the assessment of Pareto-efficiency. Intuitively, equation (3) states that the project $\mathbf{z}$ should be designed to generate the largest possible net benefit or "budget surplus" $M(U^0)$, which should be entirely redistributed to the n individuals. Here, the Pareto-utility frontier is given by the utility levels U^0 satisfying the net benefit criterion $M(U^0)=0$ (Graham, 1992, p. 836). This corresponds to $\sum_{j\in\mathbf{J}}\gamma_j[\sum_{i\in\mathbf{I}}(\mathbf{z}_{1ifj}+\beta\mathbf{z}_{2ifj})-C_j(\mathbf{z}_j)]=0$, which is the complementary slackness condition for the aggregate budget constraint (1).

Propositions 1 and 2 establish dual formulations of Pareto-efficiency.[8] Graham's focus was on developing criteria for identifying Pareto-efficient projects. The use of shadow prices for contingent claims (i.e., the use of the dual formulation) provided an elegant way to develop these criteria. Here, we are interested in exploring the risk-related components of total value. In particular, we focus attention on the role of both private and public learning in valuation under uncertainty. The primal formulation presented in proposition 1 provides an intuitively appealing framework in which to explore the concepts found in the OV and QOV literatures.

The Individual's Decision Problem

Graham (1992) simplified private goods to be contingent claims to dollars. To explore the role of private decisions and the interactions between private and public learning, we add more structure to the specification of both private goods and private information. Consider a particular individual affected by the public project, who makes private decisions under temporal uncertainty and in the absence of complete risk markets. The individual faces an allocation of private goods as well as public goods. She/he has a two-period planning horizon. At each period t, the individual makes decisions concerning the private goods represented by the vector $\mathbf{x}_t$, $t=1,2$. As above, the project $\mathbf{z}$ includes both the provision of public goods and their financing in each period, where $\mathbf{z}_{tp}$ is a vector of public goods (or bads) provided by the project in period t, and $\mathbf{z}_{tf}$ is a vector representing the financing scheme of the project in period t. The project can then be denoted by $\mathbf{z}=(\mathbf{z}_1,\mathbf{z}_2)=[(\mathbf{z}_{1p},\mathbf{z}_{1f}),(\mathbf{z}_{2p},\mathbf{z}_{2f})]$, where $\mathbf{z}_t=(\mathbf{z}_{tp},\mathbf{z}_{tf})$ is the vector of public decisions made at time t, $t=1,2$.

Uncertainty facing the individual is represented by the random variables $\mathbf{e}=(\mathbf{e}_1,\mathbf{e}_2)$, where $\mathbf{e}_t$ is assumed to become observable at the end of the tth period. The vector $\mathbf{e}$ can include any number of sources of uncertainty, including income, preferences, prices, the weather, etc. The individual has a subjective probability distribution about the random variables $(\mathbf{e}_1,\mathbf{e}_2)$ and is a Bayesian learner. First-period decisions are based on the subjective joint probability $f(\mathbf{e}_1,\mathbf{e}_2|\mathbf{x}_1,\mathbf{z}_1)=f_1(\mathbf{e}_1)f_2(\mathbf{e}_2|\mathbf{e}_1,\mathbf{x}_1,\mathbf{z})$, where $f_1(\mathbf{e}_1)$ is the marginal prior probability function of $\mathbf{e}_1$, and $f_2(\mathbf{e}_2|\mathbf{e}_1,\mathbf{x}_1,\mathbf{z}_1)$ is the posterior probability function of $\mathbf{e}_2$, conditional on the realized value of $\mathbf{e}_1$ and on $(\mathbf{x}_1,\mathbf{z}_1)$. Between time $t=1$ and $t=2$, learning takes place and the random variables $\mathbf{e}_1$ become known. In this context, learning is defined as observation of the realized values of the random variables $\mathbf{e}_1$. The second-period decisions are based on the posterior probability function $f_2(\mathbf{e}_2|\mathbf{e}_1,\mathbf{x}_1,\mathbf{z}_1)$. The posterior probability of $\mathbf{e}_2$ can reflect active learning, where both private $(\mathbf{x}_1)$ and public $(\mathbf{z}_1)$ decisions made at time $t=1$ can influence how much information is learned about $\mathbf{e}_2$ before the second-period decisions $\mathbf{x}_2$ are made. Passive learning is a special case where the posterior probability of $\mathbf{e}_2$ is independent of $(\mathbf{x}_1,\mathbf{z}_1)$ and becomes $f_2(\mathbf{e}_2|\mathbf{e}_1)$.

Intuitively, the more correlated are the random variables $\mathbf{e}_1$ and $\mathbf{e}_2$, the more informative are the signals $\mathbf{e}_1$ concerning the uncertainty $\mathbf{e}_2$. At one extreme is the perfect information case, where a perfect correlation between $\mathbf{e}_1$ and $\mathbf{e}_2$ would imply that observing the signals $\mathbf{e}_1$ provides perfect information about the realized value of the random

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[8] Whereas proposition 2 assumes convexity, note that proposition 1 holds more general conditions. See Luenberger for a good discussion of these issues.

variables $\mathbf{e}_2$. At the other extreme, if $\mathbf{e}_1$ and $\mathbf{e}_2$ are independent random variables, observing the signals $\mathbf{e}_1$ is completely uninformative about the actual values taken by the random variables $\mathbf{e}_2$. Intermediate situations allow for partial learning. The most general case is that of partial learning, with some of the learning being active and some passive.

In general, the random variables in $(\mathbf{e}_1, \mathbf{e}_2)$ are correlated with public information $\mathbf{e}^{\mathrm{I}} = (\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})$. The quality of information provided by observing random variables in $\mathbf{e}^{\mathrm{I}}$ (e.g., as measured by their degree of correlation with $\{\mathbf{e}_i, i \in \mathbf{I}\}$) is then dependent on both private and public decisions. The information linkages can work both ways: public information can affect private decisions, and private information can affect public decisions. For example, public information about the quality of groundwater can affect private decisions about housing location and the use of water supplies. In the other direction, private information can either remain private (e.g., private landowners not reporting the presence of endangered species on their property) or become public (e.g., fishermen or birdwatchers reporting changes in the abundance of various species), which in either case may influence the design of public policy. For example, information about individuals' health problems associated with exposure to pollutants can help motivate government action.

The individual's preferences are represented by the von Neumann–Morgenstern utility function $u(y, \mathbf{x}_1, \mathbf{x}_2, \mathbf{z}, \mathbf{e}_1, \mathbf{e}_2)$. As stated in proposition 1, we assume *nonsatiation* with respect to income ($u(y, \cdot)$ is a strictly increasing function of y).[9] We also assume that $u(\cdot)$ is finite for all $(\mathbf{e}_1, \mathbf{e}_2)$ and for all feasible values of $(y, \mathbf{x}_1, \mathbf{x}_2, \mathbf{z})$, and that the individual behaves in a way consistent with the expected utility hypothesis.[10] The feasible sets for the private decisions $\mathbf{x}_1$ and $\mathbf{x}_2$ are denoted by $\mathbf{X}_1 = \mathbf{X}_1(\mathbf{z}_1)$ and $\mathbf{X}_2 = \mathbf{X}_2(\mathbf{x}_1, \mathbf{z}_1, \mathbf{z}_2)$, respectively: $\mathbf{x}_1 \in \mathbf{X}_1$ and $\mathbf{x}_2 \in \mathbf{X}_2$. This allows public goods to influence the range of possibilities for the individual's private decisions (e.g., publicly funded cancer research). It can also represent dynamics as current decisions influence future feasibility. Private economic decisions are then made in a sequential decision framework as follows

$$(5) \qquad U(y, \mathbf{z}) = \operatorname*{Max}_{\mathbf{x}_1 \in \mathbf{X}_1} \mathbf{E}_1 \left\{ \operatorname*{Max}_{\mathbf{x}_2 \in \mathbf{X}_2} \mathbf{E}_2 \times [u(y, \mathbf{x}_1, \mathbf{x}_2, \mathbf{z}, \mathbf{e}_1, \mathbf{e}_2)] \right\}$$

where $\mathbf{E}_t$ is the expectation operator based on the subjective information available at time t, and $U(y, \mathbf{z})$ is the indirect objective function. The optimal solution to equation (5) is denoted by $\mathbf{x}_1^*(y, \mathbf{z})$ and $\mathbf{x}_2^*(y, \mathbf{z}, \mathbf{e}_1)$. In the case of active learning, some of the $\mathbf{x}_1$ decisions would include information-gathering activities that can be used to reduce future uncertainty. In this case, the utility function would take the form $u(\cdot) = u(y - c(\mathbf{x}_1), \mathbf{x}_1, \mathbf{x}_2, \mathbf{z}, \mathbf{e}_1, \mathbf{e}_2)$, where $c(\mathbf{x}_1)$ is the cost of obtaining and processing information associated with information gathering-activities in $\mathbf{x}_1$.

The Valuation of Risk

In this section, we seek to isolate the valuation of risk in the individual's sequential decision problem (5). One approach to determining the value of risk is to analyze the individual's willingness to pay (WTP), or willingness to accept compensation (WTA), for alternative risky situations. *Ex ante* monetary compensations have been proposed as measures of the implicit cost of private risk bearing by both Pratt and Arrow in the context of static risk, and by Lavalle in the context of temporal risk.[11] These compensation measures are in general not unique. In the presence of income effects, it is well known that WTP and WTA are not identical. This applies both to welfare analysis in general and to the valuation of risk in particular, and suggests that there is more than one possible measure of the value of risk. In what follows, we propose a particular *ex ante* monetary measure of the value of risk that allows us to decompose the value of risk into two additive components: one that reflects the value of learning over time, and one that reflects (static) risk preferences.

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[9] We implicitly assume that, under nonsatiation, the individual budget constraint has been substituted into the objective function using a numeraire good. As a result of this substitution, income risk and price risk appear directly in the individual's utility function (as reflected by **e**).

[10] The expected utility hypothesis is not required for our analysis. Our results could be derived using a nonexpected utility model (i.e., a model that is "nonlinear" in the probabilities; see Machina 1987). This includes the case of prospect theory, as discussed by Kahneman and Tversky and Smith (1992). Note that, in this context, the issue of dynamic inconsistency may arise (see Machina 1989).

[11] We focus on *ex ante* valuation for two reasons: it is the appropriate approach to assess the value of information and learning (see Lavalle; Hanemann 1989); and it provides the basis for evaluating Pareto-efficiency under uncertainty (Graham 1981, 1992). A discussion of *ex ante* versus *ex post* welfare evaluation is presented in Starr, Harris, Milne and Shefrin, and Hammond.

Working from equation (5), define the *conditional value of risk* as the *ex ante* value V that satisfies the implicit equation:

$$\text{(6a)}\quad U(y,\mathbf{z}) = U^{\mathrm{L}}(y+V,\mathbf{z})$$

where

$$\text{(6b)}\quad U^{\mathrm{L}}(y+V,\mathbf{z}) = \operatorname*{Max}_{\substack{\mathbf{x}_1\in\mathbf{X}_1\\ \mathbf{x}_2\in\mathbf{X}_2}}\{u(y+V),\mathbf{x}_1,\mathbf{x}_2,\mathbf{z}, \mathbf{E}_1(\mathbf{e}_1),\mathbf{E}_1(\mathbf{e}_2)\}$$

and $U(y,\mathbf{z})$ is the indirect objective function defined in equation (5). The notation $U^{\mathrm{L}}(\cdot)$, which we will use again below, denotes the indirect objective function for an optimization problem in which *all risk has been removed* by replacing the random variables $(\mathbf{e}_1,\mathbf{e}_2)$ with their *ex ante* means $((\mathbf{E}_1(\mathbf{e}_1),\mathbf{E}_1(\mathbf{e}_2))$. Equation (6b) characterizes a solution where all decisions are made *in the absence of all risk* (and thus in the absence of learning). The value V defined in equation (6a) can be interpreted as the selling value of risk, which is the smallest *ex ante* amount of money the individual is willing to accept (or willing to pay if negative) to have $\mathbf{e}_1$ and $\mathbf{e}_2$ equal to their mean values with probability one.

The function $V(y,\mathbf{z})$ measures the individual's implicit value of private risk bearing, conditional on the project $\mathbf{z}$. This value can be decomposed into two additive parts, one related to learning, and one related to risk preferences. The role of learning can be isolated by comparing the sequential problem (5) to an open-loop problem in which learning is ignored. Define the *conditional selling value of information*[12] as the *ex ante* value S satisfying the following implicit equation[13]

$$\text{(7)}\quad U(y,\mathbf{z}) = \operatorname*{Max}_{\substack{\mathbf{x}_1\in\mathbf{X}_1\\ \mathbf{x}_2\in\mathbf{X}_2}}\{\mathbf{E}_1[u(y+S,\mathbf{x}_1,\mathbf{x}_2,\mathbf{z},\mathbf{e}_1,\mathbf{e}_2)]\}$$

where $U(y,\mathbf{z})$ is the indirect objective function defined in equation (5). The implicit solution of equation (7) is denoted by $S(y,\mathbf{z})$: it depends in general on exogeneous income y and is conditional on the project $\mathbf{z}$. It can be interpreted as the *gross* selling value of the information that becomes available between $t=1$ and $t=2$. It measures the smallest *ex ante* amount of money the individual is willing to receive to choose both $\mathbf{x}_1$ *and* $\mathbf{x}_2$ without the benefit of learning $\mathbf{e}_1$, and can be thought of as the individual's valuation of the ability to maintain flexible plans and react to new information as it becomes available. It is well established in the literature that the *gross* value of information is always nonnegative: $S(y,\mathbf{z})\geq 0$ (Lavalle).[14] This result follows directly from the convexity of the maximum operator and Jensen's inequality. $S(y,\mathbf{z})=0$ only in situations where the individual either faces a completely inflexible position or does not expect to learn anything of value.[15]

When information is costless, $S(y,\mathbf{z})$ is also the *net* value of information (which is always nonnegative because additional costless information cannot make the decision maker worse off). When information is costly, the *net* value of private information is $S(y,\mathbf{z})-c(\mathbf{x}_1)$, where $c(\mathbf{x}_1)$ is the cost of private information. Information is worth obtaining only when its gross value, $S(y,\mathbf{z})$, is greater than the cost of obtaining and processing it, $c(\mathbf{x}_1)$. In the case where $S(y,\mathbf{z})<c(\mathbf{x}_1)$ for all feasible $\mathbf{x}_1$, there is no incentive for active learning: the agent would choose all information gathering activities in $\mathbf{x}_1$ to be zero.

Risk preferences can also be isolated in this framework. Define the *conditional risk aversion premium* as the *ex ante* value R satisfying the following implicit equation

$$\text{(8)}\quad U(y,\mathbf{z}) = \operatorname*{Max}_{\substack{\mathbf{x}_1\in\mathbf{X}_1\\ \mathbf{x}_2\in\mathbf{X}_2}}\{u(y+S(y,\mathbf{z})-R,\mathbf{x}_1,\mathbf{x}_2, \mathbf{z},\mathbf{E}_1(\mathbf{e}_1),\mathbf{E}_1(\mathbf{e}_2)\}$$

[12] As shown by Lavalle, the selling value of information differs from the bid value of information in the presence of income effects. Here, we rely on the selling value for two reasons. First, it is intuitively appealing because the "informed situation" typically corresponds to the status quo. Second, the selling value has the advantage of being transitive in the presence of several alternatives (Hause; Chipman and Moore).

[13] Note that, in contrast with equation (6b), the random variables $\mathbf{e}_1$ and $\mathbf{e}_2$ have *not* been set equal to their *ex ante* means in equation (7).

[14] This contrasts with the "static" value of information (e.g., Foster and Just), where information is valued at the time it is obtained. In this case, the "static" value of information can be either positive or negative depending on whether the new information is "good news" or "bad news."

[15] Under differentiability, an approximate value of S is given by

$$S \cong -2\mathrm{tr}\{[\partial x_2^*/\partial\mathbf{e}_1]'[\partial^2 u/\partial\mathbf{x}_2^2][\partial\mathbf{x}_2^*/\partial\mathbf{e}_1]\mathrm{Var}_1(\mathbf{e}_1)\}/[\partial u/\partial y],$$

where $\mathrm{tr}\{\cdot\}$ denotes the trace, and $\partial u/\partial y>0$ under nonsatiation. Assuming an interior solution to equation (5), this expression for S is obtained by taking Taylor series approximations of equation (7) in the neighborhood of the riskless case (where $\mathbf{e}_1=\mathbf{E}_1(\mathbf{e}_1)$ and $\mathbf{e}_2=\mathbf{E}_1(\mathbf{e}_2)$). Note that this approximation is consistent with the general nonnegativity of the conditional value of information $S(y,\mathbf{z})$: the matrix $[\partial^2 u/\partial\mathbf{x}_2^2]$ is negative semidefinite from the second-order condition for an interior solution to the maximization problem (5).

where $U(y, \mathbf{z})$ is the indirect objective function in equation (5). Equation (8) defines R as the individual's *ex ante* WTP (or WTA if negative) to replace the random variables $(\mathbf{e}_1, \mathbf{e}_2)$ by their *ex ante* means $(\mathbf{E}_1(\mathbf{e}_1), \mathbf{E}_1(\mathbf{e}_2))$ under the following two conditions: (i) there is no learning; and (ii) the individual receives the ex ante monetary compensation $S(y, \mathbf{z})$ defined in equation (7). The value R can be interpreted as the private cost of the risk $(\mathbf{e}_1, \mathbf{e}_2)$ in the absence of learning: it measures the largest *ex ante* amount of money the individual is willing to pay to face a riskless world, assuming that no learning takes place.

The sign of the risk aversion premium $R(y, \mathbf{z})$ depends on the risk preferences of the individual, which are represented by the curvature of $u(\cdot)$. The individual is said to be risk averse, risk neutral, or risk loving depending on whether R is positive, zero or negative, respectively (Arrow; Pratt; Duncan). A sufficient condition for the individual to exhibit risk aversion (risk loving behavior) is that his/her utility function is concave (convex) in $(\mathbf{e}_1, \mathbf{e}_2)$. The individual is risk neutral if $u(\cdot)$ is linear in $(\mathbf{e}_1, \mathbf{e}_2)$.[16]

The conditional value of risk defined by equation (6) satisfies

$$(9) \qquad V(y, \mathbf{z}) = S(y, \mathbf{z}) - R(y, \mathbf{z})$$

where $S(y, \mathbf{z})$ and $R(y, \mathbf{z})$ are defined in (7) and (8), respectively. Equation (9) shows that the total value of risk $V(y, \mathbf{z})$ can be decomposed into two additive parts: the value of information $S(y, \mathbf{z}) \geq 0$, minus the risk aversion premium $R(y, \mathbf{z})$.[17] Because R can be positive, zero, or negative depending on the nature of the individual's risk preferences, $V(y, \mathbf{z})$ can also take any sign. Without additional information on the nature of risk preferences, we cannot sign the total value of risk a priori. Even under risk aversion (where $R > 0$), the value of risk $V(y, \mathbf{z})$ can be positive (when $S > R$), zero ($S = R$), or negative ($S < R$).

Individual Project Evaluation

The model developed above focused on the valuation of risk for a particular individual affected by the public project $\mathbf{z}$. We now turn our attention to the individual's valuation of the project itself. Again, recall that the project may be partially or fully irreversible. Let the utility function of the ith individual be $u_i(y_i, \mathbf{x}_{1i}, \mathbf{x}_{2i}, \mathbf{z}, \mathbf{e}_{1i}, \mathbf{e}_{2i})$, where y_i is the ith individual's exogenous income at time $t = 1, \mathbf{x}_{ti}$ is the ith individual's private decisions made at time t, and $\mathbf{e}_i = (\mathbf{e}_{1i}, \mathbf{e}_{2i}), i = 1, \ldots, n$. Permitting the random vector $\mathbf{e}_i = (\mathbf{e}_{1i}, \mathbf{e}_{2i})$ to be individual-specific allows for asymmetric information, where each individual can observe different signals $\mathbf{e}_{1i}$. Also, denote the feasible sets for $\mathbf{x}_{1i}$ and $\mathbf{x}_{2i}$ by $\mathbf{X}_{1i}$ and $\mathbf{X}_{2i}$, respectively, which allows for heterogeneous preferences and technology across individuals.

Individual welfare effects can be analyzed using either bid price (compensating variation and/or surplus) or selling price (equivalent variation and/or surplus) versions of *ex ante* income compensation tests. In the presence of income effects, the bid price and the selling price will, in general, differ (Hicks; Hause; Willig), and may differ for other reasons as well (Kahneman and Tversky; Gregory; Knetsch; Hanemann, 1991; Boyce et al.). Although the model can be developed using either the bid price or the selling price, we focus our attention on the selling price because it has the advantage of preserving a transitive welfare ranking of alternatives, whereas the bid price could provide an intransitive ranking if the project involves more than two alternatives (Hause; Chipman and Moore).

Let $\mathbf{z} = \mathbf{z}^w$ if the project is undertaken and $\mathbf{z}^0$ in the absence of the project. For the ith individual, the *selling price* is defined as the *ex ante* amount of money P_i^s which implicitly satisfies

$$(10) \quad U_i(y_i, \mathbf{z}^w) = U_i(y_i + P_i^s, \mathbf{z}^0)$$

where $U_i(y_i, \mathbf{z}^w)$ denotes the ith individual's indirect objective function in equation (5)

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[16] Under differentiability, an approximate value for R is given by

$$R \cong -2\mathrm{tr}\{[\partial^2 u/\partial(\mathbf{e}_1, \mathbf{e}_2)^2]\ \mathrm{Var}_1(\mathbf{e}_1, \mathbf{e}_2)\}/[\partial u/\partial y],$$

where $\mathrm{tr}\{\cdot\}$ denotes the trace, and $\partial u/\partial y > 0$ under nonsatiation. This expression for R is obtained by taking Taylor series approximations of equation (8) in the neighborhood of the riskless case (where $\mathbf{e}_1 = \mathbf{E}_1(\mathbf{e}_1)$ and $\mathbf{e}_2 = \mathbf{E}_1(\mathbf{e}_2)$). Note that this approximation is consistent with the analysis of risk premia presented by Arrow, Pratt, Duncan, and others: the risk premium $R(y, \mathbf{z})$ is, respectively, positive, zero, or negative when the individual's utility function $u(\cdot)$ is concave, linear, or convex in $(\mathbf{e}_1, \mathbf{e}_2)$.

[17] Whereas focusing on the risk neutral case, Fisher and Hanemann (1990, p. 403) suggest that their analysis could be extended to the risk averse case by "including a risk-premium term". Our analysis provides this extension by showing formally how both the conditional value of information and the conditional risk premium affect valuation.

with the project $\mathbf{z}^w$. The reference situation is given on the left-hand side of equation (10) by the *ex ante* utility level obtained under the project $\mathbf{z}^w$. Equation (10) defines $P_i^s(y_i, \mathbf{z}^w, \mathbf{z}^0)$ as the ith individual's minimum WTA to forego the project.

To examine the role of risk in individual welfare evaluation, use equations (6) and (9) to rewrite equation (10) in terms of the associated *riskless* problem

$$(11)\quad U_i^{\mathrm{L}}[y_i + S_i(y_i, \mathbf{z}^w) - R_i(y_i, \mathbf{z}^w), \mathbf{z}^w] = U_i^{\mathrm{L}}[y_i + S_i(y_i, \mathbf{z}^0) - R_i(y_i, \mathbf{z}^0) + P_i^s, \mathbf{z}^0]$$

where $S_i(y_i, \mathbf{z}^w)$ denotes the value of private information with the project, $S_i(y_i, \mathbf{z}^0)$ is the value of private information without the project, $R_i(y_i, \mathbf{z}^w)$ denotes the private cost of risk aversion with the project, $R_i(y_i, \mathbf{z}^0)$ is the private cost of risk aversion without the project, and $U_i^{\mathrm{L}}[\cdot]$ is defined in equation (6b) as the ith individual's indirect utility function in the absence of risk, but after compensation $(S_i - R_i)$ is received (or paid if negative) to move from the sequential problem with learning to the riskless problem.

Working from the left-hand side of equation (11), define B_i^s as the *ex ante* amount of money satisfying

$$(12)\quad U_i^{\mathrm{L}}[y_i + S_i(y_i, \mathbf{z}^w) - R_i(y_i, \mathbf{z}^w), \mathbf{z}^w] = U_i^{\mathrm{L}}[y_i + S_i(y_i, \mathbf{z}^w) - R_i(y_i, \mathbf{z}^w) + B_i^s, \mathbf{z}^0]$$

where $B_i^s(y_i, \mathbf{z}^w, \mathbf{z}^0)$ can be interpreted as the ith individual's selling value of the project in the absence of risk (more will be said about B_i^s shortly). Under nonsatiation, substituting equation (12) in equation (11) implies

$$\begin{aligned}(13)\quad P_i^s(y_i,\mathbf{z}^w,\mathbf{z}^0) &= [S_i(y_i,\mathbf{z}^w) - S_i(y_i,\mathbf{z}^0)] \\ &\quad - [R_i(y_i,\mathbf{z}^w) - R_i(y_i,\mathbf{z}^0)] \\ &\quad + B_i^s(y_i,\mathbf{z}^w,\mathbf{z}^0) \\ &= \Delta S_i - \Delta R_i + B_i^s\end{aligned}$$

where $\Delta S_i = S_i(y_i, \mathbf{z}^w) - S_i(y_i, \mathbf{z}^0)$, and $\Delta R_i = R_i(y_i, \mathbf{z}^w) - R_i(y_i, \mathbf{z}^0)$.

For the ith individual, equation (13) decomposes the total value of the project into three additive components. The first term in (13), ΔS_i, measures the *impact of the project on the value of private information.* If the project increases the ability of the ith individual to respond to new information, then $S_i(y_i, \mathbf{z}^w) > S_i(y_i, \mathbf{z}^0)$ and ΔS_i is positive, thus contributing to an increase in the selling price P_i^s. Alternatively, if the project either reduces or eliminates the ith individual's ability to adjust to future information (e.g., permanent injuries due to the project), $S_i(y_i, \mathbf{z}^w) < S_i(y_i, \mathbf{z}^0)$ and ΔS_i is negative. Loss of flexibility corresponds to decreased welfare, and this will lower the value of the project, all else equal. Note that in general we can expect $\Delta S_i < 0$ when the project has irreversible effects on private decisions. This is true whether or not the project itself is irreversible. For example, pollution of a body of water may be reversible, but the health effects for consumers may be long term and irreversible.

The second term on the right-hand side of (13), $-\Delta R_i$, measures the *impact of the project on the conditional risk aversion premium.* For the ith individual, ΔR_i can be interpreted as the change in the cost of risk aversion. For a risk averse individual, ΔR_i can be positive, negative or zero, depending on whether she/he faces more or less uncertainty as a result of the project. If the project reduces the uncertainty faced by the individual (e.g., by bolstering the population of an endangered species), then the risk aversion premium would fall. If the project increases uncertainty (e.g., by moving an endangered species closer to extinction), the risk aversion premium would rise. All else equal, a project that lowers (heightens) risk would be worth more (less) to a risk averse individual.

Using equation (9), the first two terms on the right-hand side of equation (13) can be rewritten as

$$(14)\quad \Delta S_i - \Delta R_i = V_i(y_i, \mathbf{z}^w) - V_i(y_i, z^0) \equiv \Delta V_i$$

where $V_i(y_i, \mathbf{z}^w)$ is the total value of risk with the project and $V_i(y_i, \mathbf{z}^0)$ is the total value of risk without the project. Equation (14) states that the change in the total private value of risk, ΔV_i, equals the change in the private value of information, ΔS_i, minus the change in the private cost of risk aversion, ΔR_i. This reflects the fact that the project may alter the individual's risk exposure, as well as his/her ability to learn.

Equations (13) and (14) also imply that $B_i^s = P_i^s - \Delta V_i$. Because P_i^s measures the total value of the project and ΔV_i measures its risk benefit (or cost, if negative), it follows that $B_i^s(y_i, \mathbf{z}^w)$ can be interpreted as the *nonrisk* *benefit* (or cost, if negative) of the project for the ith individual. Indeed, $B_i^s(y_i, \mathbf{z}^w)$ is the exact measure of the ith individual selling price for the project if the project has no net effect on the risks faced by the individual, $\Delta V_i = 0$. This includes as a special case the situation where the ith individual lives in a riskless world, with $V_i = 0$. Thus, in the absence of risk, B_i^s could be obtained by traditional welfare measures (e.g., consumer surplus).[18] Alternatively, whenever $\Delta V_i = \Delta S_i - \Delta R_i \neq 0$, it follows that $P_i^s \neq B_i^s$. In such cases, ignoring risk provides a biased measure of the welfare impact of the project. Because uncertainty is pervasive, this provides a framework for understanding the various roles of risk in project evaluation.

Aggregate Valuation and Efficient Resource Allocation under Risk

Having defined a welfare measure of the project for the ith individual, it remains to aggregate this measure over the n individuals affected by the project. Proposition 1 provides a criterion to choose a Pareto-efficient project, based on individual welfare measures P_i^s. The optimization problem in (2) assumes that public learning $\mathbf{e}_1^{\mathrm{I}}$ occurs between periods 1 and 2, and that second-period public decisions $\mathbf{z}_2(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})$ can make use of this learning. We are interested in isolating the welfare gains associated with retaining the flexibility to make more informed second-period decisions about both the provision of public goods and their financing. To do so, compare the sequential decision in (2) to an open-loop solution where all decisions are made based on the information available at time $t = 1$ (i.e., conditional only on e_0^{I}). Define the *conditional value of public information* obtained between $t = 1$ and $t = 2$ as

$$
\begin{aligned}
(15)\quad & V^{\mathrm{I}}(\mathbf{z}_1, \alpha) \\
&= \operatorname*{Max}_{\mathbf{z}_2(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})} \left\{ \sum_{i \in \mathbf{I}} \alpha_i P_i^s(y_i, \mathbf{z}_{1if} - \beta \mathbf{z}_{2if}, \mathbf{z}_p) : \text{s.t. } \mathbf{z}_p \in \mathbf{Z} \text{ and } (1) \right\} \\
&\quad - \operatorname*{Max}_{\mathbf{z}_2(\mathbf{e}_0^{\mathrm{I}})} \left\{ \sum_{i \in \mathbf{I}} \alpha_i P_i^s(y_i, \mathbf{z}_{1if} - \beta \mathbf{z}_{2if}, \mathbf{z}_p) : \text{s.t. } \mathbf{z}_p \in \mathbf{Z} \text{ and } (1) \right\}.
\end{aligned}
$$

Let $\mathbf{z}_2^c(\mathbf{z}_1, \alpha, \mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})$ be the solution of the first optimization problem in equation (15), and $\mathbf{z}_2^a(\mathbf{z}_1, \alpha, \mathbf{e}_0^{\mathrm{I}})$ be the solution of the second optimization problem. Whereas both solutions depend on the first-period decisions $\mathbf{z}_1$, they differ by the amount of public information available: the decision rule $\mathbf{z}_2^c(\cdot)$ depends on $(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})$, whereas the decision rule $\mathbf{z}_2^a(\cdot)$ depends only on $(\mathbf{e}_0^{\mathrm{I}})$, which corresponds to the absence of public learning. $V^{\mathrm{I}}(\mathbf{z}_1, \alpha)$ is the difference between the weighted aggregate WTA obtained from choosing the second-period allocation $\mathbf{z}_2$ with and without public learning, conditional on the first-period decisions $\mathbf{z}_1$. Because the information structure associated with $(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})$ is necessarily finer than with $\mathbf{e}_0^{\mathrm{I}}$, it follows that the decision rule $\mathbf{z}_2^c(\mathbf{z}_1, \alpha, \mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})$ is less restrictive than the decision rule $\mathbf{z}_2^a(\mathbf{z}_1, \alpha, \mathbf{e}_0^{\mathrm{I}})$ for any given $(\mathbf{z}_1, \alpha)$. This implies that $V^{\mathrm{I}}(\mathbf{z}_1, \alpha) \geq 0$. This means that people would value a project in which the $\mathbf{z}_2$ decisions were made with public learning ($\mathbf{z}_2^c$) more than a project in which the $\mathbf{z}_2$ decisions were made without public learning ($\mathbf{z}_2^a$), all else equal.

In general, $V^{\mathrm{I}}(\mathbf{z}_1, \alpha) \geq 0$ can be interpreted as the *gross* value of public information. It is also the net value of information if public information is costless. In this case, public learning will typically be efficient, implying that any open-loop solution (which neglects public learning) will be suboptimal. In the situation where information is costly, let $C_a(\mathbf{z}_1)$ denote the cost of obtaining and processing public information, where C_a depends on the public information gathering activities in $\mathbf{z}_1$. It follows that the *net* value of public information would be $V^{\mathrm{I}}(\mathbf{z}_1, \alpha) - C_a(\mathbf{z}_1)$. In the case of active learning, public learning would be worth it only when the gross value of information $V^{\mathrm{I}}(\mathbf{z}_1, \alpha)$ is greater than its cost C_a. If $V^{\mathrm{I}}(\mathbf{z}_1, \alpha) < C_a(\mathbf{z}_1)$ for all feasible $\mathbf{z}_1$, then there would be no incentive to learn: it would be efficient for the group to choose the information gathering activities in $\mathbf{z}_1$ to be zero. In the case where $V^{\mathrm{I}}(\mathbf{z}_1, \alpha) > C_a(\mathbf{z}_1)$, neglecting public learning would necessarily be suboptimal.

To illuminate the role of information, we can use equation (13) to rewrite equa-

[18] Note that, in the presence of risk, this does not imply that B_i^s will equal the expected value of "*ex post* consumer surplus". As argued by Graham (1981), the relationship between B_i^s and the expected "*ex post* consumer surplus" is unclear and largely irrelevant to the evaluation of risky projects.

tion (15) as

$$
\begin{aligned}
(16)\quad & V^{\mathrm{I}}(\mathbf{z}_1,\alpha) \\
&=\left\{\sum_{i\in\mathbf{I}}\alpha_i P_i^s[y_i,\mathbf{z}_1\mathbf{z}_2^c(\mathbf{z}_1,\alpha,\mathbf{e}_0^{\mathrm{I}},\mathbf{e}_1^{\mathrm{I}})]\right\} \\
&\quad -\left\{\sum_{i\in\mathbf{I}}\alpha_i P_i^s[y_i,\mathbf{z}_1,\mathbf{z}_2^a(\mathbf{z}_1,\alpha,\mathbf{e}_0^{\mathrm{I}})]\right\} \\
&=\sum_{i\in\mathbf{I}}\alpha_i\big\{\Delta S_i[y_i,\mathbf{z}_1,\mathbf{z}_2^c(\mathbf{z}_1,\alpha,\mathbf{e}_0^{\mathrm{I}},\mathbf{e}_1^{\mathrm{I}})] \\
&\qquad -\Delta S_i[y_i,\mathbf{z}_1,\mathbf{z}_2^a(\mathbf{z}_1,\alpha,\mathbf{e}_0^{\mathrm{I}})] \\
&\qquad -\Delta R_i[y_i,\mathbf{z}_1,\mathbf{z}_2^c(\mathbf{z}_1,\alpha,\mathbf{e}_0^{\mathrm{I}},\mathbf{e}_1^{\mathrm{I}})] \\
&\qquad +\Delta R_i[y_i,\mathbf{z}_1,\mathbf{z}_2^a(\mathbf{z}_1,\alpha,\mathbf{e}_0^{\mathrm{I}})] \\
&\qquad +B_i^s[y_i,\mathbf{z}_1,\mathbf{z}_2^c(\mathbf{z}_1,\alpha,\mathbf{e}_0^{\mathrm{I}},\mathbf{e}_1^{\mathrm{I}})] \\
&\qquad -B_i^s[y_i,\mathbf{z}_1,\mathbf{z}_2^a(\mathbf{z}_1,\alpha,\mathbf{e}_0^{\mathrm{I}})]\big\}.
\end{aligned}
$$

Our formulation of the policy maker's problem allows efficiency gains from better-informed second-period decisions concerning *both* public goods provision $\mathbf{z}_{2p}$ and the financing scheme $\mathbf{z}_{2f}$. Equation (16) provides useful insights on the potential sources of these efficiency gains. The gains can come from: (1) the impact of improved public information on the private value of information ΔS_i; (2) the influence of public learning on the private cost of risk aversion ΔR_i; and (3) the impact of better public information on the nonrisk benefits B_i^s, in the form of income effects. Our model formalizes the possible interactions between private and public decisions in the valuation of risk. Such interactions would be important whenever the public project affects the ability of individuals to respond to new information. This includes situations where the project facilitates individual use of information ($\Delta S_i > 0$), as well as situations where it hinders it ($\Delta S_i < 0$) (e.g., Dixit and Pindyck; Chavas). It is worth pointing out again that even if the project itself is partially or fully reversible, its effects on individuals' flexibility to respond to new information (as in the long-term health impacts in our groundwater example) can be significant.

In addition, even if private learning is ignored ($\Delta S_i = 0$), and if $B_i^s[y_i,\mathbf{z}_1,\mathbf{z}_2^c] - B_i^s[y_i,\mathbf{z}_1,\mathbf{z}_2^a] = 0, i \in \mathbf{I}$, then from (16), $V^{\mathrm{I}}(\mathbf{z}_1,\alpha) = -\Delta R_i[y_i,\mathbf{z}_1,\mathbf{z}_2^c] + \Delta R_i[y_i,\mathbf{z}_1,\mathbf{z}_2^a] \geq 0$. In this case, efficiency gains from public learning are generated entirely by a more efficient distribution of risk. Graham's (1981) significant contribution is that efficient state-dependent payments can lower risk exposure, or transfer risk toward less risk averse individuals, thus reducing the total cost of private risk bearing and increasing aggregate welfare gains. Here we expand on Graham's result to show that state-dependent provision of public goods can also use learning to lower risk exposure, which is a central insight of the QOV.[19]

To illuminate the role of public learning in the Pareto-efficient allocation of public resources, proposition 1 can be alternatively expressed as follows:

COROLLARY 1 (The main result). *Under nonsatiation, Pareto-efficient allocations are characterized by*

$$
\begin{aligned}
(17)\quad W^{\mathrm{I}}(\alpha) = \operatorname*{Max}_{\substack{\mathbf{z}_1(\mathbf{e}_0^{\mathrm{I}})\\ \mathbf{z}_2(\mathbf{e}_0^{\mathrm{I}})}} \Big\{ & V^{\mathrm{I}}(\mathbf{z}_1,\alpha) \\
& + \sum_{i\in\mathbf{I}}\alpha_i[\Delta S_i(\cdot) - \Delta R_i(\cdot) + B_i^s(\cdot)]: \\
& \text{s.t. } \mathbf{z}_p \in \mathbf{Z} \quad \text{and (1)}\Big\}.
\end{aligned}
$$

Proof. Substituting equation (15) into (2) yields

$$
\begin{aligned}
W^{\mathrm{I}}(\alpha) = \operatorname*{Max}_{\substack{\mathbf{z}_1(\mathbf{e}_0^{\mathrm{I}})\\ \mathbf{z}_2(\mathbf{e}_0^{\mathrm{I}})}} \Big\{ & V^{\mathrm{I}}(\mathbf{z}_1,\alpha) + \sum_{i\in\mathbf{I}}\alpha_i P_i^s \\
& \times (y_i, \mathbf{z}_{1if} - \beta\mathbf{z}_{2if}, \mathbf{z}_p): \\
& \text{s.t. } \mathbf{z}_p \in \mathbf{Z} \text{ and (1)}\Big\}.
\end{aligned}
$$

Then, using equation (13) yields the desired result.

Corollary 1 provides a useful decomposition of the role of risk in project evaluation. Equation (17) shows that welfare analysis involves *three risk components*: the public value of information $V^{\mathrm{I}}(\mathbf{z}_1,\alpha)$, the private value of information $\sum_{i\in\mathbf{I}}\alpha_i\Delta S_i(\cdot)$, and the cost of risk aversion $-\sum_{i\in\mathbf{I}}\alpha_i\Delta R_i(\cdot)$. The term $V^{\mathrm{I}}(\mathbf{z}_1,\alpha)$ is the adjustment needed in a benefit–cost analysis to induce Pareto-efficient first-period decisions in the event that the project does not incorporate public learning. Given $V^{\mathrm{I}}(\mathbf{z}_1,\alpha) \geq 0$, it follows from equation (17) that ignoring the potential for

[19] Although he does not elaborate on it, Graham (1992) briefly mentions (p. 826) that the provision of public goods can be state dependent. In our model, the choice of public goods is treated explicitly as state dependent by modeling it as a choice among *decision rules* mapping states into public decisions.

learning new information is often suboptimal in public decision making. It is clearly suboptimal when relevant public information is costless. It is also suboptimal under active learning whenever the gross value of public information is greater than its cost.

The above formulation provides useful insights into the role of public information. The conditional value of public information $V^{\mathrm{I}}(\mathbf{z}_1, \alpha)$ can be affected in various ways by the first-period decisions $\mathbf{z}_1$. First, $V^{\mathrm{I}}(\mathbf{z}_1, \alpha)$ could be increasing in $\mathbf{z}_1$. This could happen when $\mathbf{z}_1$ are information gathering activities, or when $\mathbf{z}_1$ involves research activities generating new technological opportunities for period two. In such situations, public learning provides incentive to increase $\mathbf{z}_1$. Alternatively, neglecting public learning would lead to "underinvestment" in $\mathbf{z}_1$. Second, $V^{\mathrm{I}}(\mathbf{z}_1, \alpha)$ could be independent of $\mathbf{z}_1$. In this case, neglecting public learning would have no effect on the first-period decision $\mathbf{z}_1$. Finally, $V^{\mathrm{I}}(\mathbf{z}_1, \alpha)$ could be decreasing in $\mathbf{z}_1$. This could happen when the public project involves irreversible first-period decisions that restrict the public ability to respond to future information. In such situations, public learning provides an incentive to decrease $\mathbf{z}_1$. Alternatively stated, neglecting public learning would lead to "overinvestment" in $\mathbf{z}_1$. This latter case has been the main thrust of the QOV literature.

Finally, note that this model can also be used to evaluate whether a particular project would pass the potential Pareto-improvement criterion. From equation (2) or (17), a project is said to pass the potential Pareto-improvement criterion if it is feasible, and if it is associated with decision rules $\mathbf{z}_1(\mathbf{e}_0^{\mathrm{I}})$ and $\mathbf{z}_2(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})$ such that $\sum_{i\in\mathbf{I}} \alpha_i P_i^s[y_i, \mathbf{z}_1(\mathbf{e}_0^{\mathrm{I}}), \mathbf{z}_2(\mathbf{e}_0^{\mathrm{I}}, \mathbf{e}_1^{\mathrm{I}})] > 0$. This allows for "weighted" (where the (α_i's differ across individuals) as well as "unweighted" welfare analysis.

Reuniting Weisbrod's Children

We have developed an *ex ante* welfare measure of the total value of a public project. Key features of this measure are: it allows for both private and social learning, it incorporates risk preferences, it covers both the provision of public goods and their financing, and it is state dependent. We now show that this measure incorporates the insights of both the option value and the quasi-option value literatures. The option value literature has focused on whether individuals, faced with a valuation decision characterized by uncertainty and irreversibility, would be willing to pay more or less than the expected value of their consumer surplus for a public project. Option value came to be defined as the difference between option price (OP, the maximum sure (i.e., state-independent) payment an individual would be willing to make to obtain the public goods in question), and the expected value of consumer surplus, E(CS): OP = E(CS) + OV. The OV models all either implicitly or explicitly assume a "timeless" world where there is no learning. In our model, the absence of learning implies $\Delta S_i = 0$ and $V^{\mathrm{I}} = 0$. This leaves $P_i^s = B_i^s - \Delta R_i$, which suggests that $(-\Delta R_i)$ plays a role similar to OV: they both account for the influence of risk preferences on welfare valuation. Consistent with the conclusions of the OV literature, ΔR_i can take any sign, even for under risk aversion, because the project can either increase or decrease the individual's exposure to risk (Bishop 1988).

Graham (1981) extended the analysis of risk by allowing for state-dependent project financing, which may entail efficiency gains through monetary risk redistribution. Graham's willingness-to-pay locus is formed by the set of all contingent (or state-dependent) payments that leave the individual indifferent, *ex ante*, between having and not having the project. Efficient schemes are those that maximize the aggregate sure net willingness to pay. Under convexity assumptions, this is equivalent to establishing competitive markets for contingent claims. In this context, a risk averse individual would want to redistribute risk across states by agreeing to contracts that reduce his/her risk exposure. Alternatively, if all individuals are identical and face the same risks, there is no benefit from risk redistribution, and the optimal payment scheme remains the state-independent OP.

Graham's article generated a lot of interest and some confusion. We briefly attempt to clarify three areas of confusion or misinterpretation in the subsequent literature. First, some authors have advanced the claim that the expected value of the fair-bet point (E(FB$_i$), the point on the individual's willingness-to-pay locus with the highest expected value) is the appropriate welfare measure for cases involving individual risks (Cory and Colby Saliba; Colby and Cory).

Whereas Graham has shown that this is correct if individual risks are insurable, this is not true in general. In the absence of fair insurance markets, individuals would not be willing to pay $E(FB_i)$ because they cannot purchase contingent claims that allow them to reach FB_i (Bishop 1986; Smith 1990). To the extent that insurance markets are typically incomplete, this suggests that $E(FB_i)$ will in general *not* be the appropriate welfare measure.

Second, it has been argued that the mere existence of a set of contingent contracts that could raise enough revenue to cover the cost of the project in each state is sufficient for passing a potential Pareto-improvement test (Freeman 1991, 1993). Such arguments suggest that contingent payments and compensations need not be actually collected or received. This is clearly erroneous when the state-dependent payments themselves are the source of some of the net benefits. Because the risk redistribution benefits of the state-dependent payments are solely the product of the financing scheme, not collecting state-dependent payments would eliminate the associated benefits of risk redistribution.

Third, some authors felt that Graham had advocated counting hypothetical contingent payments in welfare analyses (e.g., Ready). We believe that this is a misinterpretation of Graham's work. In fact, we feel that Ready and Graham *agree* on the role of risk in project evaluation: that any risk redistribution that actually occurs as a result of project financing should be incorporated into welfare measures. Ready provides a welfare measure that captures the benefits of risk redistribution from any state-dependent financing that actually occurs. Our model is also consistent with the view that *ex ante* valuation of risk redistribution benefits due to actual financing should be incorporated in welfare analysis.

Just as state-dependent financing can increase the value of a project, so can state-dependent provision of public goods. In a sequential decision framework, learning allows decisions about the second-period provision of public goods to be state dependent. Reversible first-period decisions preserve this flexibility, whereas completely irreversible first-period decisions restrict the second-period decisions to be state independent (e.g., Arrow and Fisher). The main thrust of the QOV literature is that flexibility to respond to new information is valuable, and project analysis should incorporate the value of this flexibility. By allowing the $\mathbf{z}_{2p}$ decisions to be state dependent, our model captures the value of learning as it relates to the provision of public goods. By explicitly considering the role of learning in the provision of public goods, we present a logical integration of QOV and Pareto-efficiency under uncertainty.

One of our contributions is that we are able to distinguish explicitly between the influence of the project on private flexibility (ΔS_i) and public flexibility (V^{I}). ΔS_i is a welfare measure of the impact of the project on the ith individual's ability to respond to new information. V^{I} is the (*ex ante*) collective social gain resulting from the public decision maker's ability to make informed and flexible second-period decisions. Whereas QOV authors have focused on the role of flexibility and irreversibility at the social level (e.g., Arrow and Fisher; Conrad 1980), others have noted that such a role applies as well at the private level (e.g., the effects of sunk costs on investments; see Dixit and Pindyck). However, nowhere have we found an exposition of the role of flexibility in both private and public decisions. Our analysis shows that both components can play a role in benefit–cost analysis. Considering both effects within a single model is informative, as evidenced by the fact that even if $V^{\mathrm{I}} = 0, \Delta S_i$ need not equal zero.

QOV has been interpreted as a correction factor required to make efficient decisions in the event that the decision maker uses a open-loop decision criterion (Freeman 1993). It has been correctly argued that the decision maker who knows how to make an appropriate (i.e., sequential) first-period decision has no need to measure QOV (Hanemann 1989; Freeman 1993). Here, we want to stress the importance of understanding the roles of private and public flexibility. First, a clear understanding of flexibility issues can help better communicate to policy makers the importance of adaptive decision making and the consequences of neglecting learning in project design and analysis. Second, when benefit–cost analysis proceeds in piecemeal fashion, an understanding of all the components of total value allows us to see which components are not being measured in any particular analysis.

Graham (1981) has shown that option price generates a lower bound measure of aggregate welfare when state-dependent payments are feasible. To the extent that collecting state-dependent payments is often feasible (e.g., user fees, fines), using OP would underestimate the net value of a project. We have drawn on the QOV literature to show that state-independent provision of public goods generates a lower bound on aggregate welfare when state-dependent provision is possible. This relationship holds whether or not the financing scheme is state dependent. These lower-bound results are special cases of the nonnegativity of the value of public information $V^{\mathrm{I}}(\mathbf{z}_1, \alpha)$ defined in equation (15). Indeed, assuming a state-independent project implies restricting V^{I} to be at its lower bound (zero) in the aggregate net benefit function (17). Of course, a state-independent project is a (restrictive) special case of our model, in which case option price is the appropriate measure of value in the absence of learning. Thus, it is clear that neglecting either the state-dependent provision of public goods or state-dependent financing schemes would provide a lower bound measure of aggregate welfare.

Conclusion

We have developed a general model that incorporates the effects of risk preferences and temporal uncertainty into welfare valuation. This provides a basis for investigating the Pareto-efficiency of a public project, including its method of financing, under risk and learning. We show explicitly how the individuals' risk aversion premium, the individuals' private value of information, and the public value of information affect the efficiency of resource allocation. By incorporating the concepts of option value and quasi-option value within a single model, and by considering both private and public decisions, the model paints a rich picture of how each contributes to total value. Hopefully, it has also resolved some of the confusion in these literatures.

We have depicted the role of temporal learning in welfare analysis. It includes both active and passive learning, and allows for partial learning. Additionally, we have emphasized the importance of valuing information and flexibility at both the individual and aggregate levels. As discussed, even if the public project is reversible, the ramifications of individual decisions may not be reversible. Neglecting to evaluate the impacts of the project on the individual's flexibility to respond to new information would omit a potentially significant component of value.

We also show how learning and the use of new information can be incorporated in the welfare analysis of a project design and its financing. This is important when state-dependent financing can generate welfare gains by improving the efficiency of risk distribution, or when learning generates benefits through the state-dependent provision of public goods. In general, it seems crucial to design public projects to capture such learning benefits. This is relevant if the cost of generating information and of implementing a state-dependent project is low. If the cost of obtaining and processing public information is high, then implementation of a fully state-dependent project would be less realistic. However, on the basis of information costs and benefits, the challenge remains to incorporate as many contingencies as possible in the design and evaluation of the project. The implications for efficient project design apply to natural resources management, but also to a wide range of public projects that involve uncertainty and irreversibility. This stresses the need for the empirical investigation of interaction effects between learning and irreversibility in project evaluation (e.g., Fisher and Hanemann 1986b; Conrad 1997).

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